

Scenario Synthesis and Macroeconomic Risk

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(Duke University)

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Bank of Albania

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Central Banks and Macroeconomic risks

Two approaches to forecast and communicate macroeconomic risks

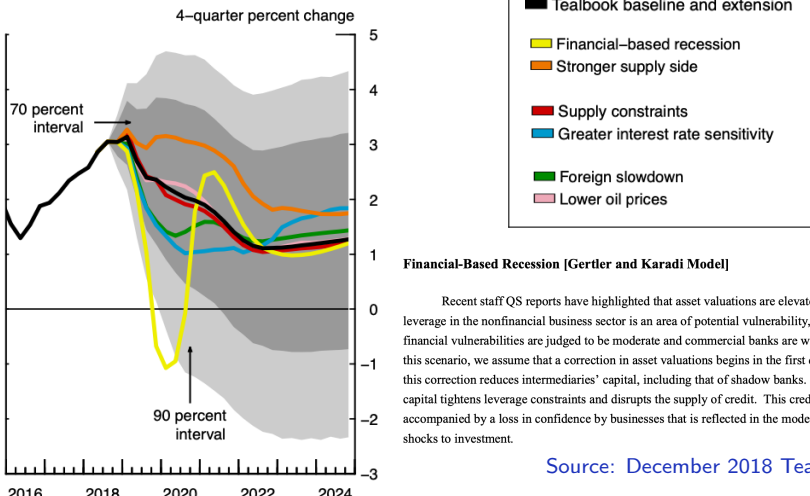
- ① Economic models that generate **baseline** and alternative **scenarios**
- ② Statistical models that generate full **predictive distributions**

Central Banks and Macroeconomic risks

(1) Economic models that generate **baseline** and alternative **scenarios**

In Tealbook since 1995

Real GDP



Central Banks and Macroeconomic risks

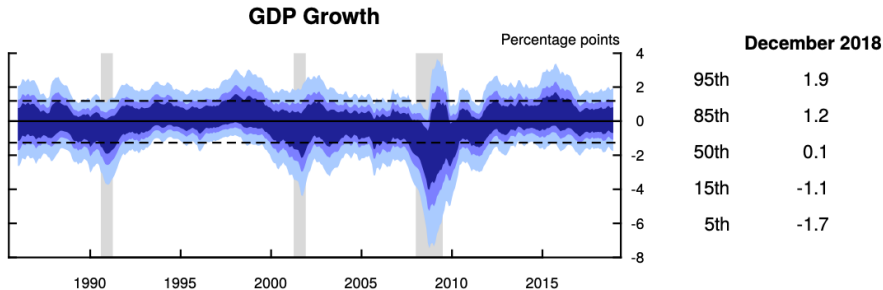
(2) Statistical models that generate full predictive distributions

Central Banks and Macroeconomic risks

(2) Statistical models that generate full predictive distributions

(a) Time-Varying Macroeconomic Risk

In Tealbook since 2017



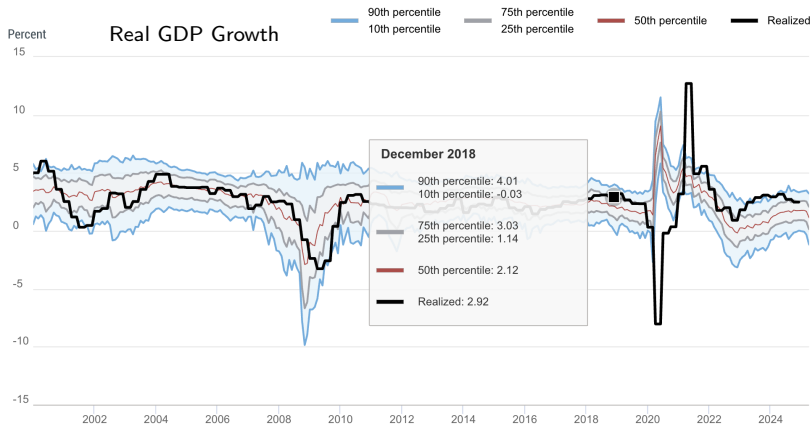
Source: December 2018 Tealbook

Central Banks and Macroeconomic risks

(2) Statistical models that generate full predictive distributions

(b) NY Fed's Outlook at Risk

Published since 2023



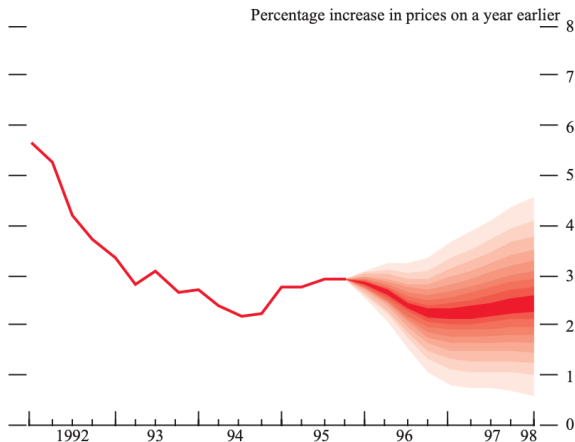
Source: NY Fed website

Central Banks and Macroeconomic risks

(2) Statistical models that generate full predictive distributions

(c) Bank of England's Fan Charts

Published since 1996



Sources: CSO and Bank of England.

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Two approaches to forecast and communicate macroeconomic risks

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- Time-Varying Macroeconomic Risk Exhibit

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Bernanke's critique: distributions are difficult to communicate

Bernanke's proposal: use scenarios

Central Banks and Macroeconomic risks

Bernanke's proposal

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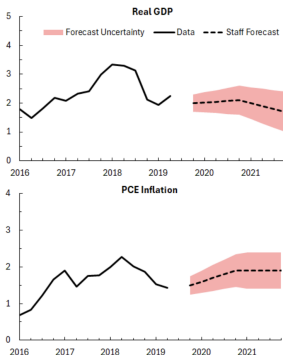
Mock-up of an Economic Review

Central Banks and Macroeconomic risks

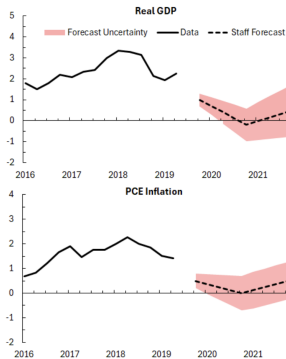
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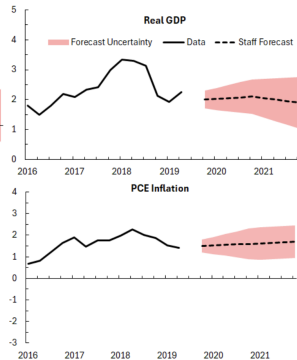
Baseline



Mild recession



Decline in infl. exp.



Source: Bernanke (2025), *Improving Fed communications: A proposal*

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- What is the probability of each scenario?
- How well the scenarios capture the macroeconomic risks?

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- What is the probability of each scenario?
- How well the scenarios capture the macroeconomic risks?

Our proposal: combine scenarios and predictive distributions

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Bernanke's critique: distributions are difficult to communicate

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What we do

- Bayesian methodology to reconciling these two approaches
 - communicative strength of narrative scenarios
 - statistical rigor of predictive models

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 - *Baseline density*
 - *Alternative scenarios*

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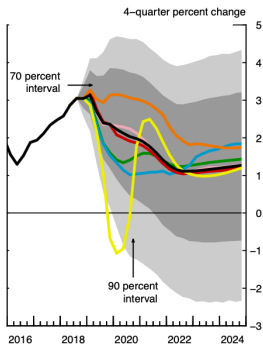
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 - Provide **probability for each scenario**
 - Assess **scenario set incompleteness**

Case study: December 2018 Tealbook

Real GDP



Baseline

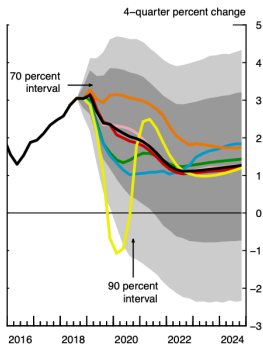
Tealbook baseline and extension

Scenarios

- Financial-based recession
- Stronger supply side
- Supply constraints
- Greater interest rate sensitivity
- Foreign slowdown
- Lower oil prices

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Real GDP



GDP Growth 2019:Q4

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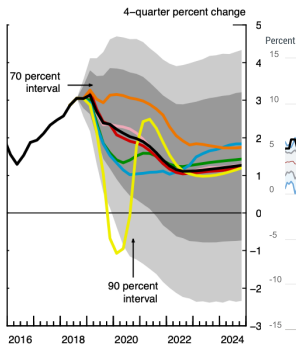
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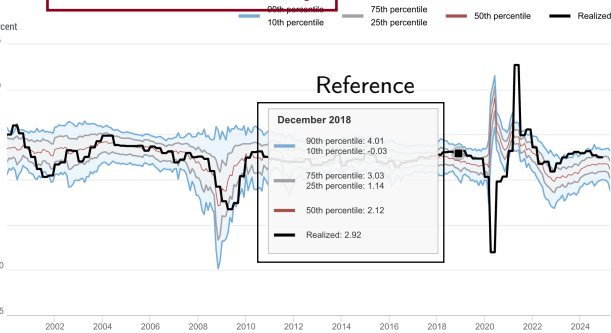
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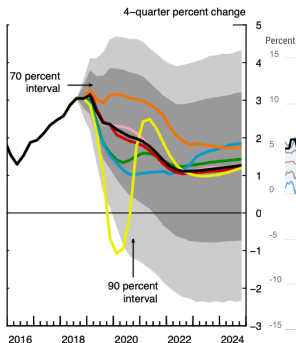
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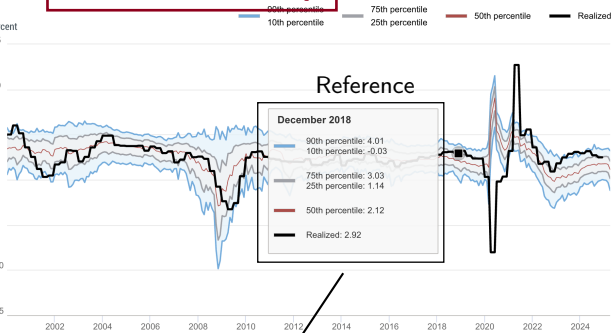
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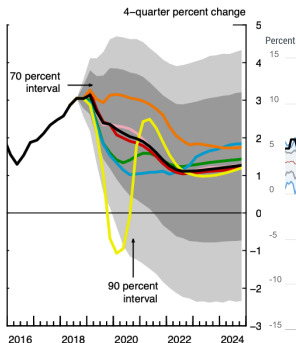
GDP Growth 2019:Q4



Synthesis

Case study: December 2018 Tealbook

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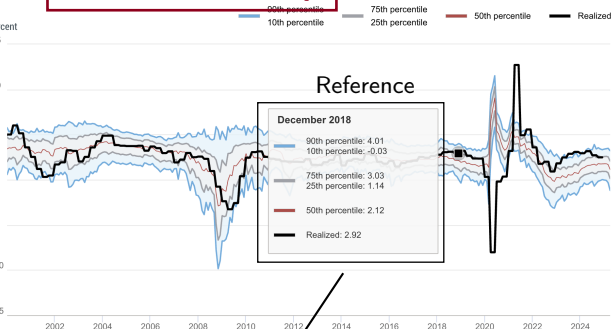
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GDP Growth 2019:Q4



Synthesis

Baseline	48%
Stronger S.S.	21%
G.I.R.S.	13%
For. Slow.	15%
Recession	4%

Incompleteness 2%

How we do it

- Construct the **synthesis**
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How we do it

- Construct the **synthesis**
 - *Baseline density* $p_0(y)$
 - *Alternative scenarios* $p_j(y), j = 1, \dots, J$
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$$f(y|\alpha) = \sum_{j=0,\dots,J} \alpha_j p_j(y)$$
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$$p(y)$$

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- Baseline density*
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- that **better concord** with \implies **Expected Missclassification Rate**

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$$\pi_{pf} \equiv E[P(H_f|y)|H_p] = \int_y \frac{f(y)p(y)}{\{f(y) + p(y)\}} dy$$

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Because we are optimizing a convex function over the probability simplex

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- **Good**: can handle several scenarios
- **Bad**: shrinkage is unstable when scenarios are close to one another

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December 2018 Tealbook

Tealbook Baseline and scenarios

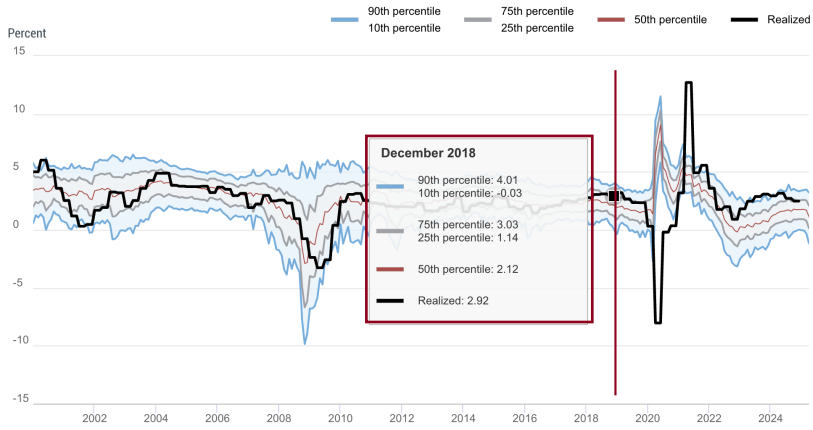
Outlook at Risk Reference

Year-over-Year GDP growth forecast for 2019:Q4

The Reference

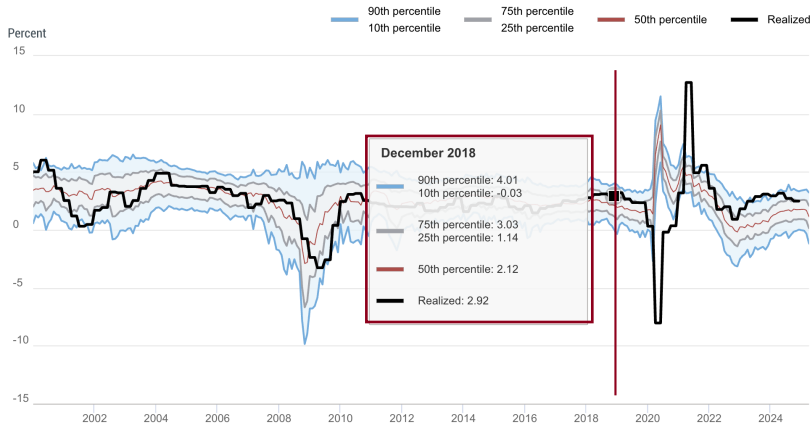
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(a) NY Fed's Outlook at Risk



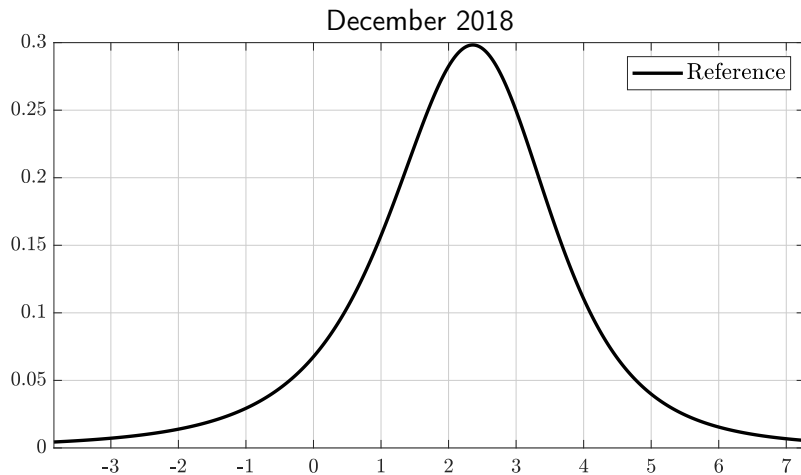
The Reference

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- Fit a skew t-distribution on these percentiles

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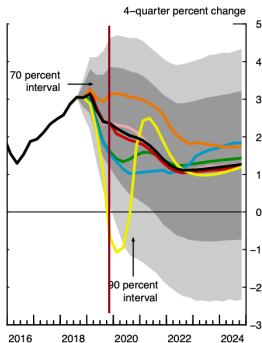


The Baseline

The Baseline

December 2018

Real GDP

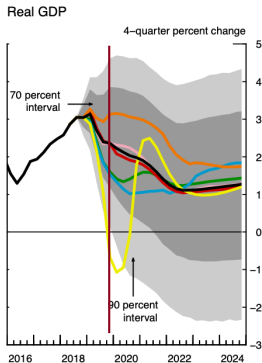


December 2018

j	Scenario S	P15	P50	P85
0	Baseline	1.2	2.4	3.9

The Baseline

December 2018

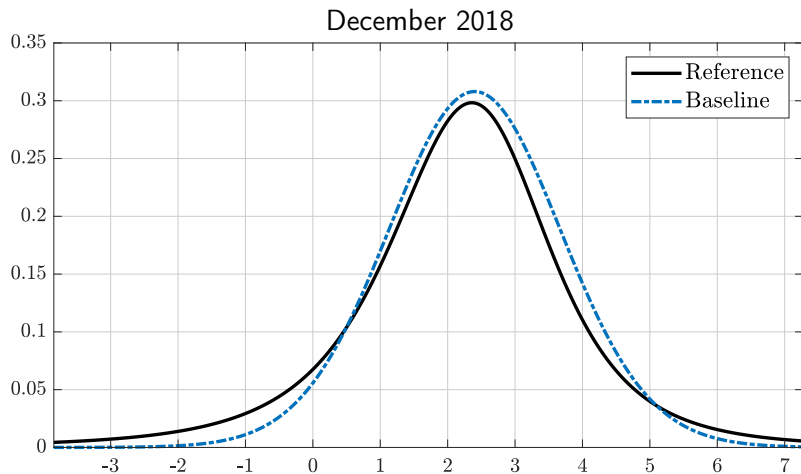


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Baseline Density \Rightarrow Fit a t-distribution with 50 DF

The Baseline

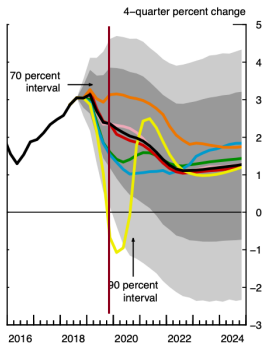


The Scenarios

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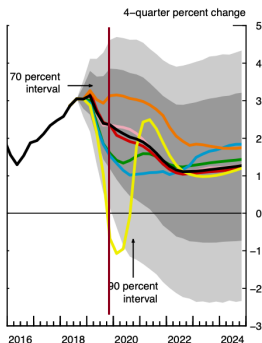
Real GDP



The Scenarios

December 2018

Real GDP

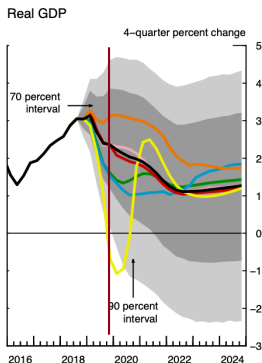


December 2018

<i>j</i>	Scenario <i>S</i>	P15	P50	P85
0	Baseline	1.2	2.4	3.9
1	Financial based recession		-0.6	
2	Stronger supply side		3.1	
3	Supply constraints		2.4	
4	Greater interest rate sensitivity		1.5	
5	Foreign slowdown		1.6	

The Scenarios

December 2018



December 2018

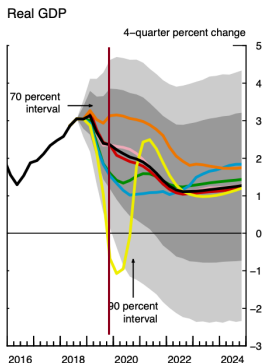
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Paper: Tilt the baseline distribution to get the scenario densities

▶ ET

The Scenarios

December 2018



December 2018

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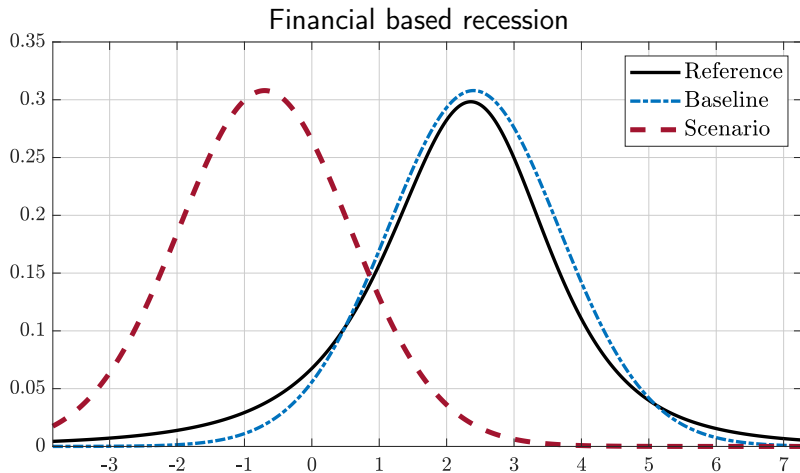
Paper: Tilt the baseline distribution to get the scenario densities

► ET

Today: Shift the baseline distribution to meet scenario p50

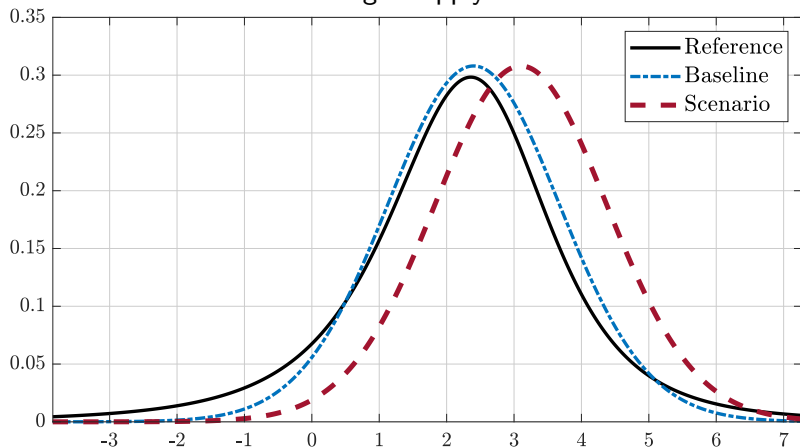
► SD

The Scenarios

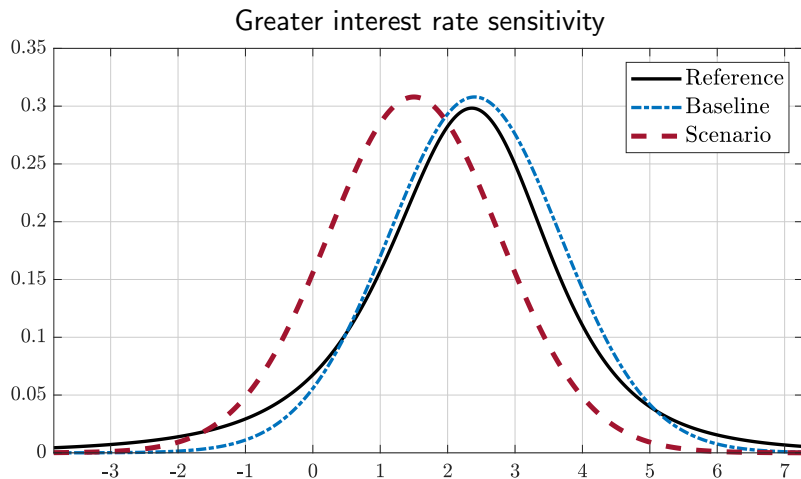


The Scenarios

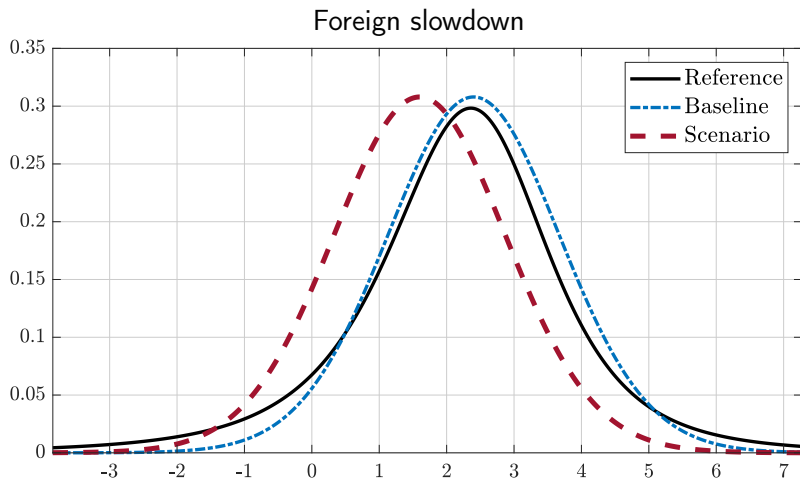
Stronger supply side



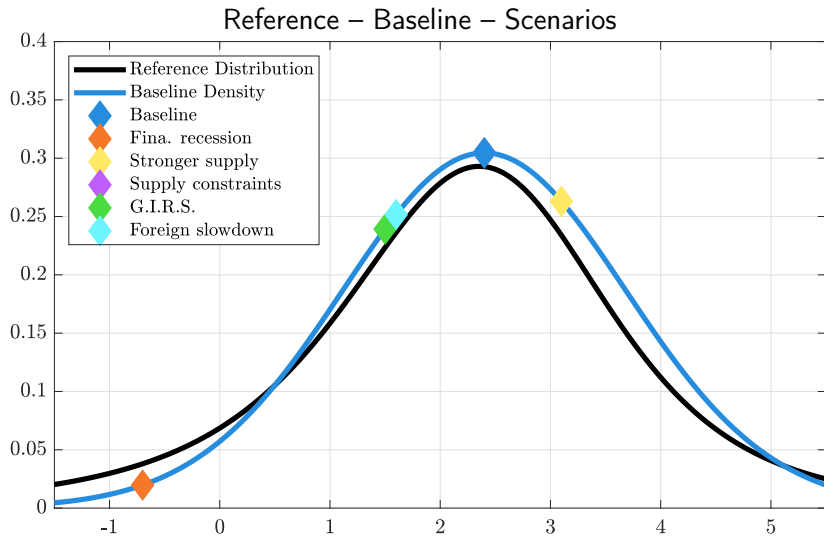
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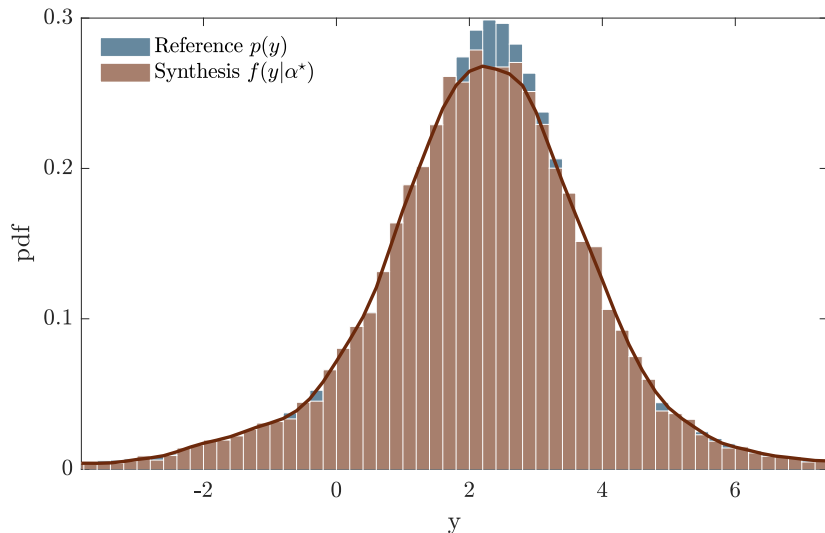


Scenario Synthesis

2018 Tealbook

Scenario Synthesis

2018 Tealbook



► step-by-step

Scenario Synthesis

2018 Tealbook

j	Scenario \mathcal{S}	ESS	EMR	$\mathbb{P}(\mathcal{S}_j)$
0	Baseline	96.8	0.50	48.4%
1	Financial-based recession	14.7	0.23	3.6%
2	Stronger supply side	73.3	0.46	20.6%
3	Supply constraints			
4	Greater interest rate sensitivity	76.5	0.46	12.7%
5	Foreign slowdown	80.7	0.47	14.7%
Synthesis		99.4	0.50	

► step-by-step

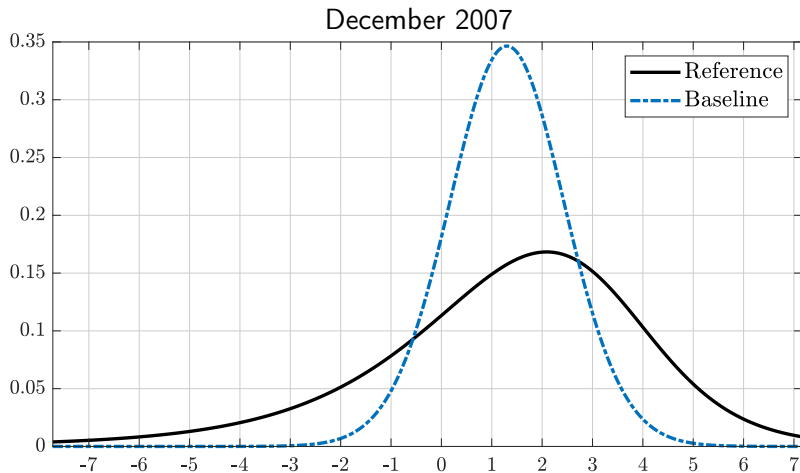
December 2007 Tealbook

Tealbook Baseline and scenarios

Outlook at Risk Reference

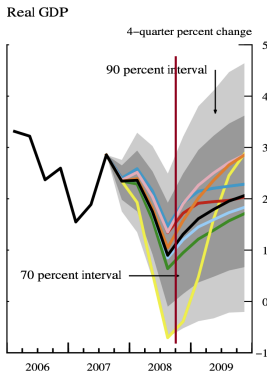
Year-over-Year GDP growth forecast for 2008:Q4

The Reference and the Baseline



The Scenarios

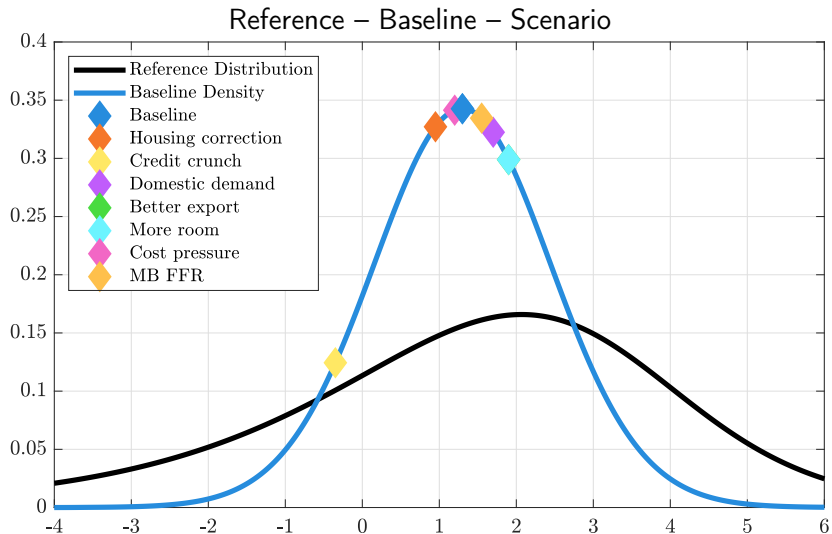
December 2007



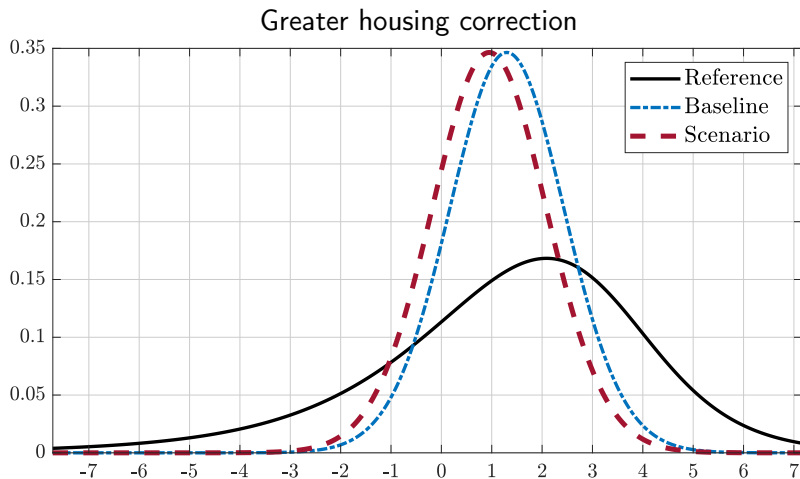
December 2018

<i>j</i>	Scenario <i>S</i>	P15	P50	P85
0	Baseline	0.1	1.3	2.5
1	Greater housing correction		0.9	
2	Credit crunch		-0.4	
3	Stronger domestic demand		1.7	
4	Better export performance		1.9	
5	More room to grow		1.9	
6	Greater cost pressure		1.2	
7	Market-based FFR		1.6	

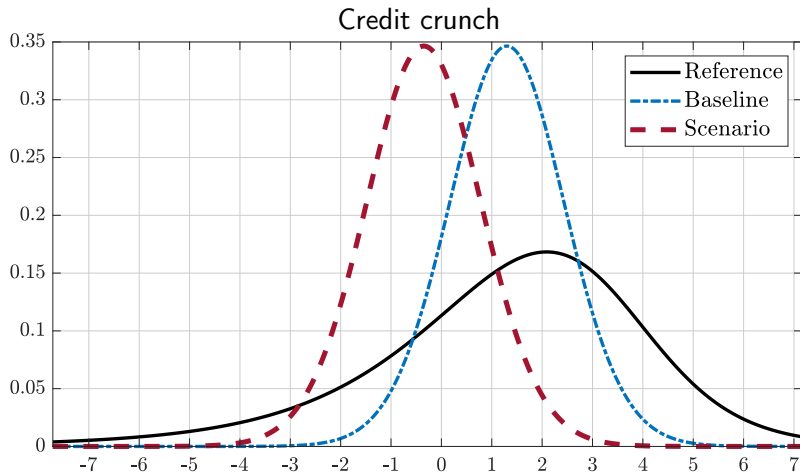
The Scenarios



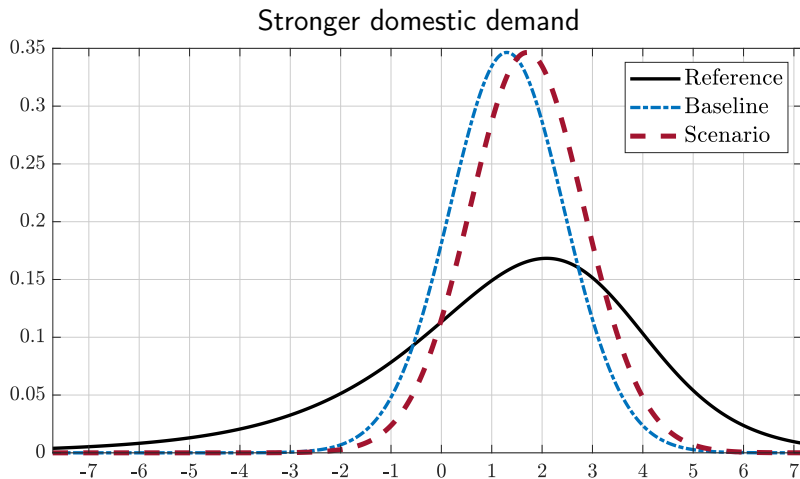
The Scenarios



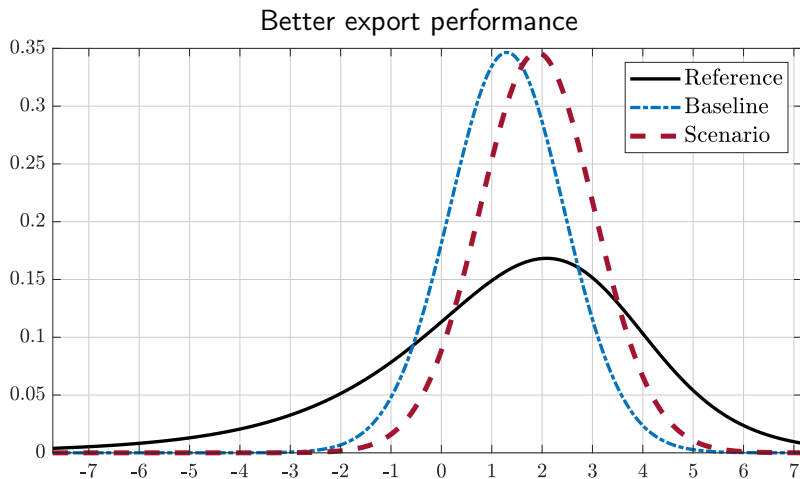
The Scenarios



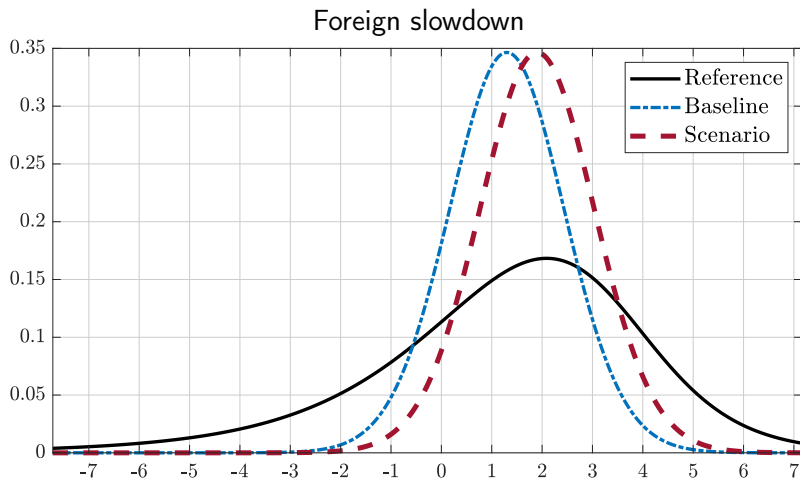
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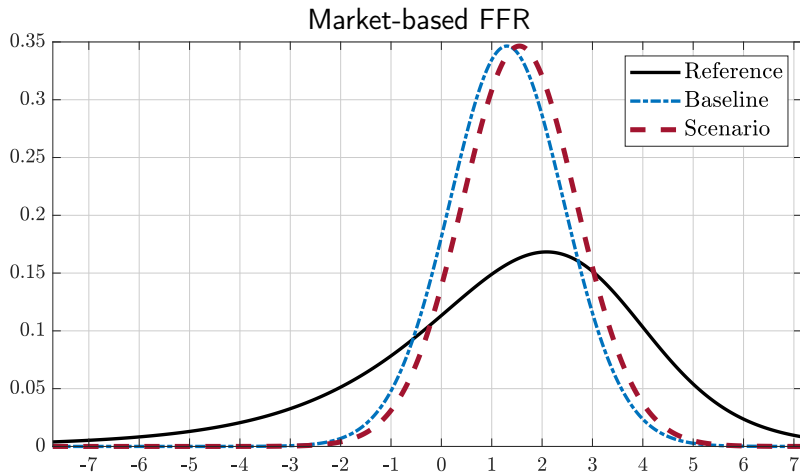
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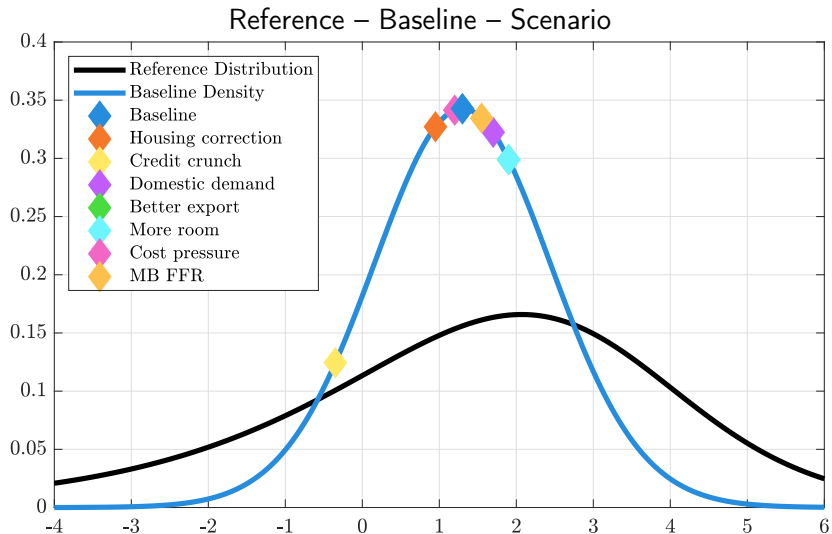
The Scenarios



The Scenarios



The Scenarios

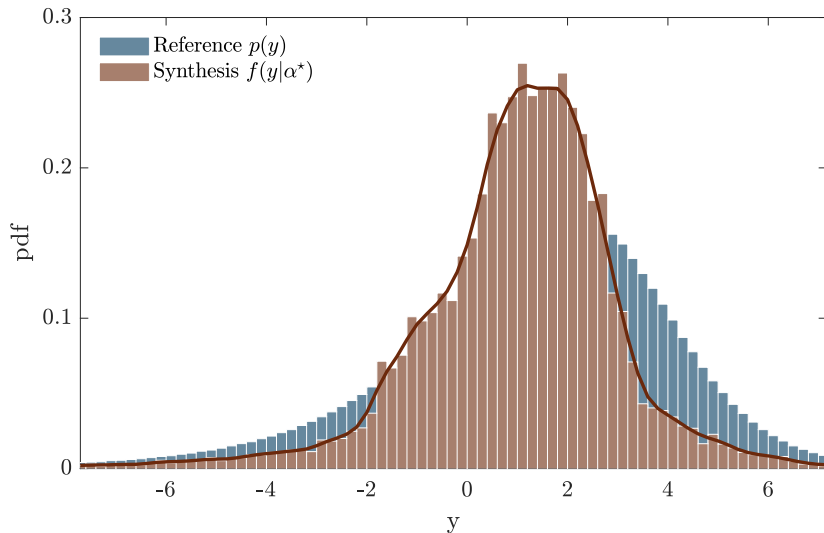


Scenario Synthesis

2007 Tealbook

Scenario Synthesis

2007 Tealbook



Scenario Synthesis

2007 Tealbook

j Scenario \mathcal{S}	ESS	EMR	$\mathbb{P}(\mathcal{S}_j)$
0 Baseline	63.5	0.43	25.1%
1 Greater housing correction	59.6	0.42	1.4%
2 Credit crunch	40.7	0.35	25.1%
3 Stronger domestic demand	66.2	0.44	18.3%
4 With better export performance	66.7	0.44	25.1%
5 More room to grow			
6 Greater cost pressure	62.5	0.43	1.5%
7 Market-based federal funds rate	65.4	0.43	3.4%
<hr/>			
Synthesis	79.0	0.46	

December 2018 Tealbook

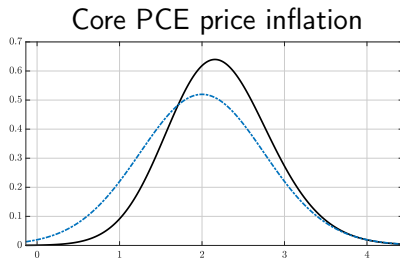
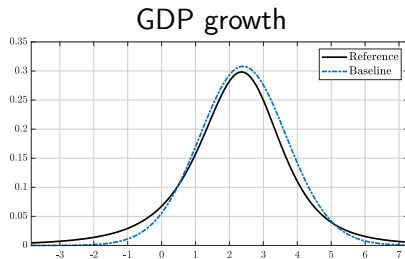
Tealbook Baseline and scenarios

Outlook at Risk Reference

Year-over-Year GDP growth & Core PCE inflation
forecast for 2019:Q4

Reference – Baseline – Scenarios

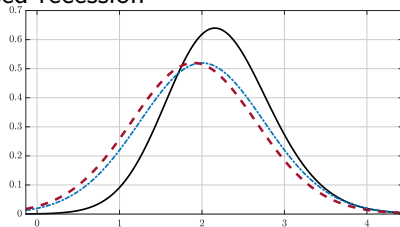
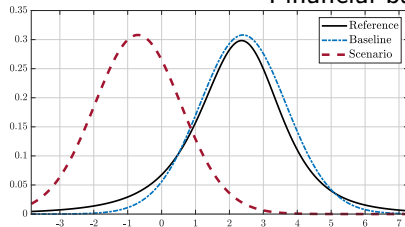
December 2018



Reference – Baseline – Scenarios

December 2018

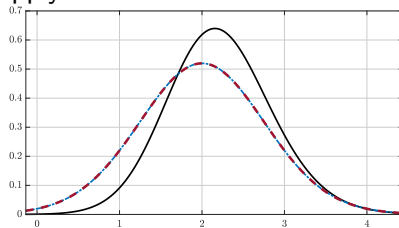
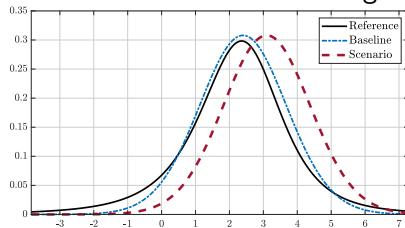
Financial based recession



Reference – Baseline – Scenarios

December 2018

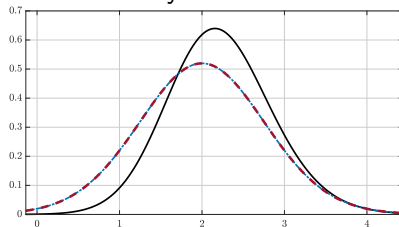
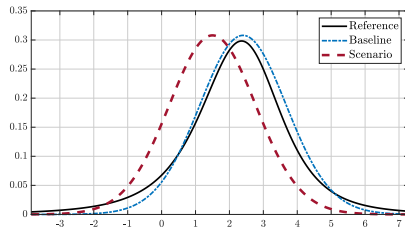
Stronger supply side



Reference – Baseline – Scenarios

December 2018

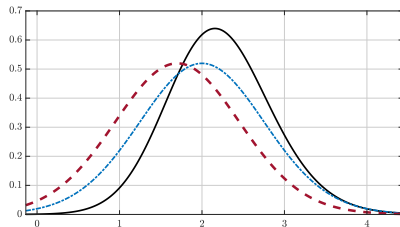
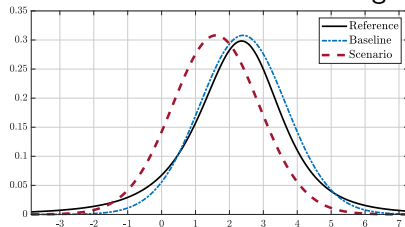
Greater interest rate sensitivity



Reference – Baseline – Scenarios

December 2018

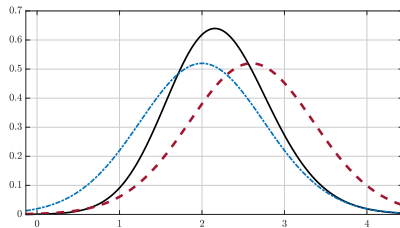
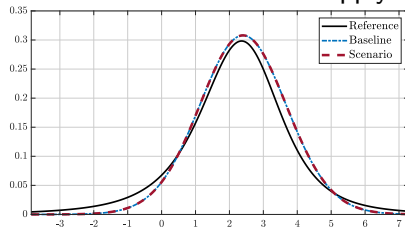
Foreign slowdown



Reference – Baseline – Scenarios

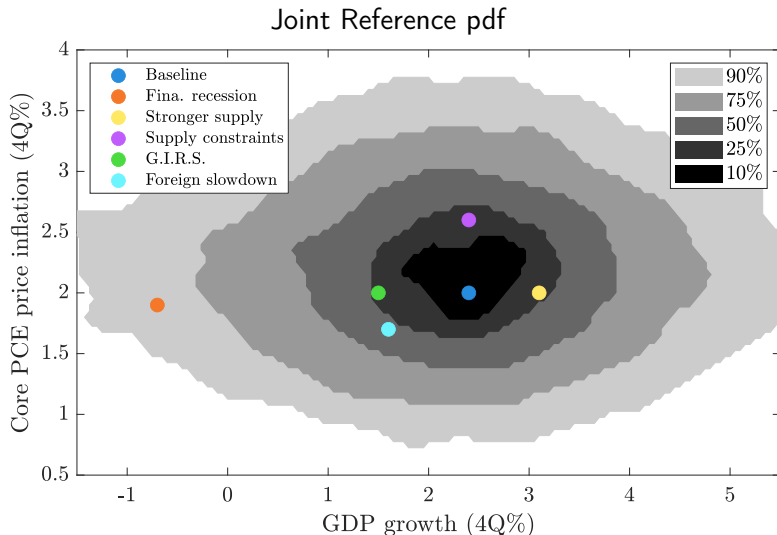
December 2018

Supply constraints



Reference – Baseline – Scenarios

December 2018

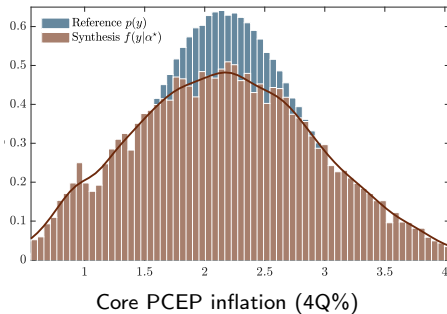
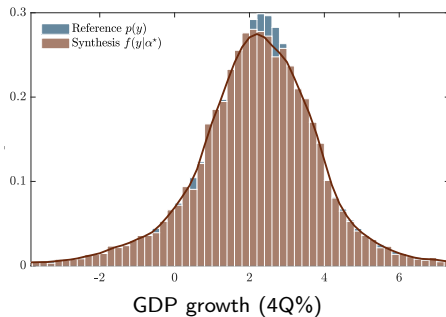


Scenario Synthesis

2018 Tealbook

Scenario Synthesis

2018 Tealbook



► step-by-step

Scenario Synthesis

2018 Tealbook

j	Scenario \mathcal{S}	ESS	EMR	$\mathbb{P}(\mathcal{S}_j)$
0	Baseline	73.8	0.48	30%
1	Financial-based recession	9.8	0.23	3%
2	Stronger supply side	55.3	0.44	16%
3	Supply constraints	62.8	0.46	30%
4	Greater interest rate sensitivity	58.8	0.45	20%
5	Foreign slowdown	28.5	0.41	2%
Synthesis		83.7	0.48	

► step-by-step

Outline

Conclusions



Conclusions

- What we did

- Bayesian Methodology to combine
 - Model-based forecasts with narrative scenarios
 - Statistical models that generate full predictive distributions

- How we did it

- Synthesis of *Baseline* and *Alternative scenarios*
- that better concord with the *Reference predictive distribution*

- Which tools we used

- Montecarlo Simulation + Importance sampling
- Entropic tilting
- VAR as a copula
- Expected miss-classification rate for concordance

- What we got

- Probability for each scenario
- Measure of scenario set incompleteness
- Discussion of Backstop scenario

Scenario Synthesis and Macroeconomic Risk

Tobias Adrian

(International Monetary Fund)

Domenico Giannone

(Johns Hopkins University)

Matteo Luciani

(Federal Reserve Board)

Mike West

(Duke University)

19th South-Eastern European Economic Research Workshop
Bank of Albania

November 6, 2025

Disclaimer: The views expressed in this paper are those of the authors and do not necessarily reflect the views and policies of the Board of Governors, the Federal Reserve System, or the International Monetary Fund, its Management, or its Executive Directors.

Our methodology step-by-step

Univariate

- 1 Sample from the Reference
 - Let y^i , ($i = 1, \dots, n$), be a **sample drawn** from $p(y)$

Our methodology step-by-step

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 - This yields a set of ET weights u_j^i

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 - Scenario synthesis IS weights: $w_f^i = \sum_{j=0, \dots, J} \alpha_j w_j^i$

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 - EMR $\pi_{pf}(\alpha) = \frac{1}{n} \sum_{i=1}^n \frac{w_f^i}{w_f^i + w_p^i}$, where $w_p^i = \frac{1}{n}$

Our methodology step-by-step

Multivariate jointly

The Reference

- Produce unconditional forecast from the NY Fed Large BVAR
- Get the marginal densities for the Reference
- Transform the VAR forecast to match the marginal of the reference
⇒ Get Joint Multivariate Reference, VAR is a copula

The Baseline and the scenarios

- Get marginal densities for the Baseline and the Scenarios
- Compute deciles for each marginal

Evaluate the Baseline and the Scenarios

- Tilt the Reference to match the deciles of each marginal
- This yields a set of Entropic Tilting weights w_j^i

Compute the optimal synthesis

- Scenario synthesis ET weights: $w_f^i = \sum_{j=0, \dots, J} \alpha_j w_j^i$
- EMR $\pi_{pf}(\alpha) = \frac{1}{n} \sum_{i=1}^n \frac{w_f^i}{w_f^i + w_p^i}$, where $w_p^i = \frac{1}{n}$

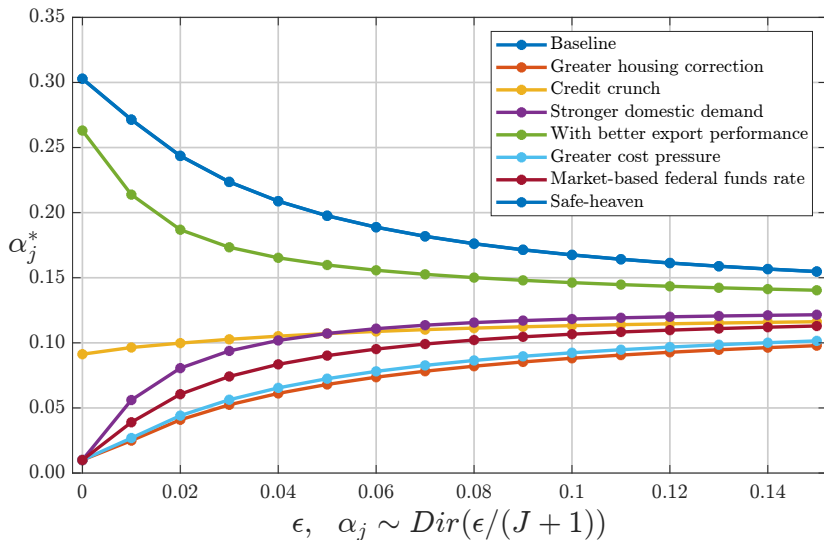
Dirichlet Prior

- Let $\sum_j \alpha_j = 1$
- Let $\mathbf{a} = (a_1, \dots, a_J)$
- Simplification $a \geq 1$

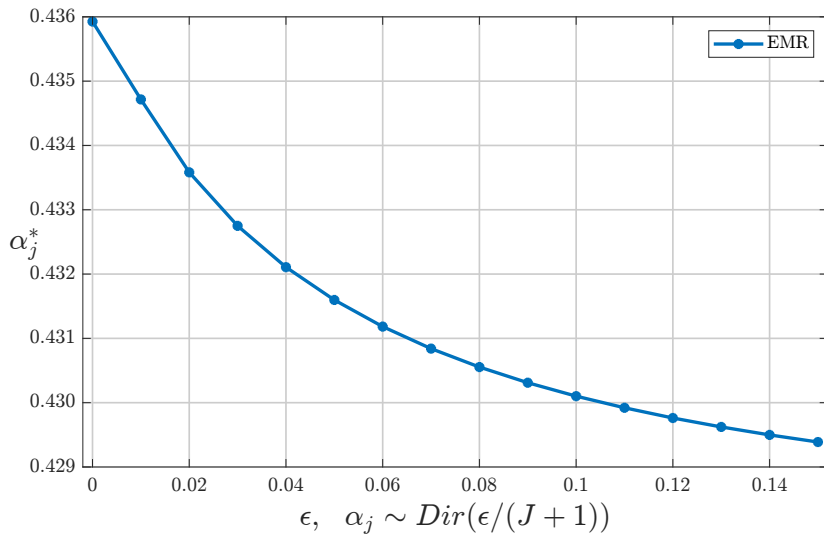
	$\alpha \sim \text{Dir}(\mathbf{a})$	$\alpha \sim \text{Dir}(1)$	$\alpha \sim \text{Dir}(1 + \epsilon)$
$E[\alpha_j]$	$\frac{a_j}{\sum_j a_j}$	$\frac{1}{J}$	$\frac{1}{J}$
Variance	$\frac{a_j(\sum_k a_k - a_j)}{(\sum_k a_k)^2(\sum_k a_k + 1)}$	$\frac{(J-1)}{J^2(J+1)}$	
Mode α_j	$\frac{a_j - 1}{\sum_j a_j - J}$	not unique	$\frac{1}{J}$

$\alpha_j \sim$	$\text{Be}(a_j, \sum_k a_k - a_j)$	$\text{Be}(1, J - 1)$	$\text{Be}(1 + \epsilon, (J - 1)(1 + \epsilon))$
$E[\alpha_j]$	$\frac{a_j}{\sum_j a_j}$	$\frac{1}{J}$	$\frac{1}{J}$
Mode	$\frac{a_j - 1}{\sum_j a_j - 2}$	0	$\frac{\epsilon}{J(1+\epsilon)-2}$

Dirichlet prior



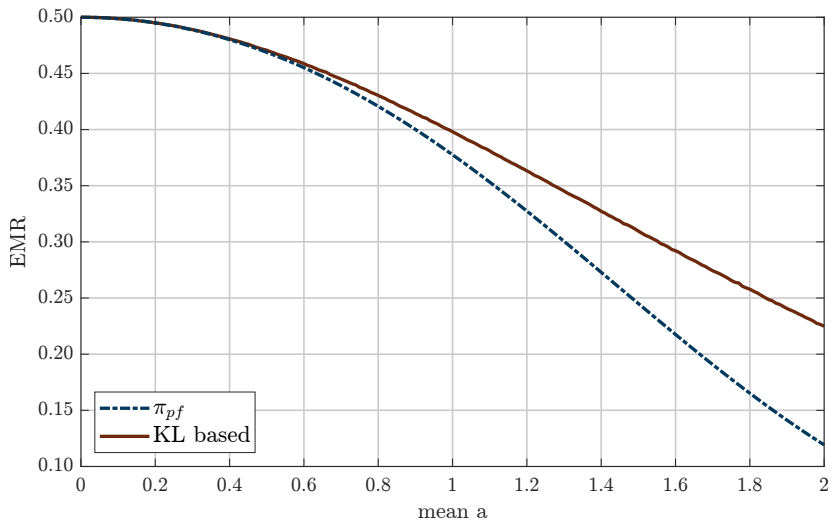
Dirichlet prior



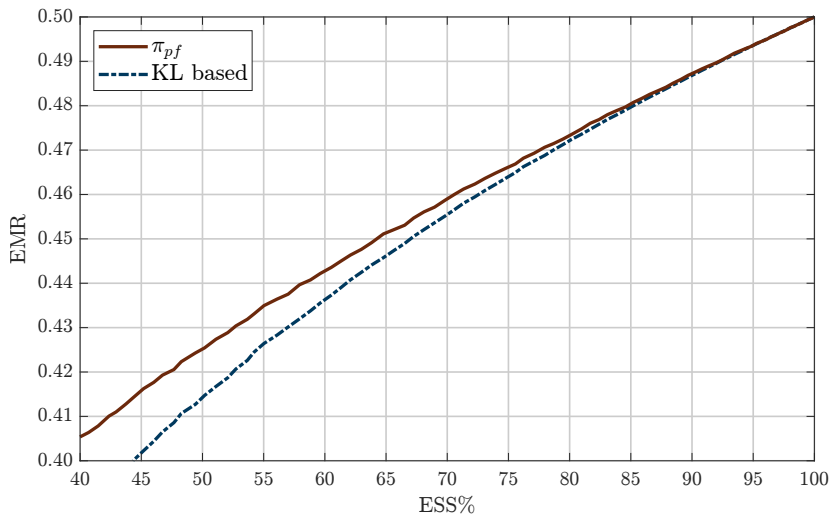
An example

- Reference $p(y) = N(0, 1)$
- Synthesis $f(y) = N(a, 1)$, $a \geq 0$.
- Questions:
 - How does EMR changes as a increases?
 - How does ESS changes as EMR changes?

An example



An example



How to get the scenario distributions?

- We rarely have full predictive distribution for the scenarios
- We usually have a point forecast $m_j = \text{median}(y|\mathcal{S}_j)$

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 $\implies p_j(y)$ is “closest to” $p_0(y)$ s.t. being consistent with m_j
- **Entropic tilting**: “closest to” = minimize Kullback-Leibler distance

$$KL_{p_0|p_j} = \int_y \log \left(\frac{p_j(y)}{p_0(y)} \right) p_j(y) dy$$

$$\tau_j^* = \arg \min \sum_{i=1}^n \exp(\tau_j' (g(y_i) - \bar{g})), \quad g(y_i) = 1[y_i \leq m_j], \bar{g} = 0.5$$

$$w_j^i = \frac{\exp(\tau_j^{*'} g(y_i))}{\arg \min \sum_{i=1}^n \exp(\tau_j^{*'} g(y_i))}$$

Scenarios Distributions

December 2018

<i>j</i>	Scenario <i>S</i>	P50	P15	P85
0	Baseline	2.4	1.2	3.9
1	Financial based recession	−0.6		
2	Stronger supply side	3.1		
3	Greater interest rate sensitivity	1.5		
4	Foreign slowdown	1.6		

► Back

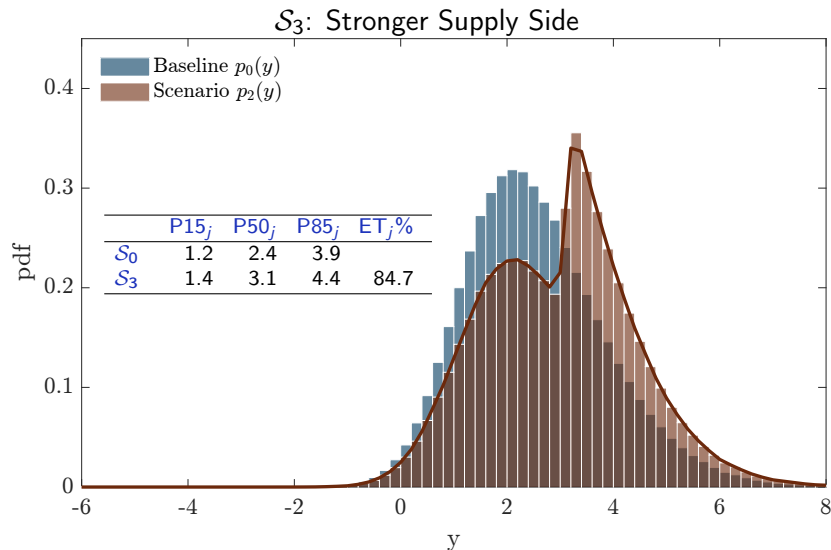
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► Back

Scenarios Distributions

[► Back](#)

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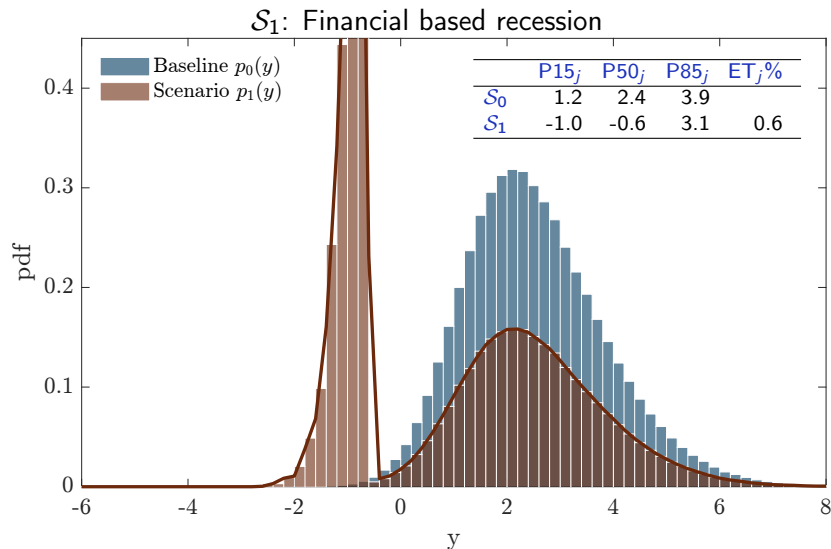
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► Back

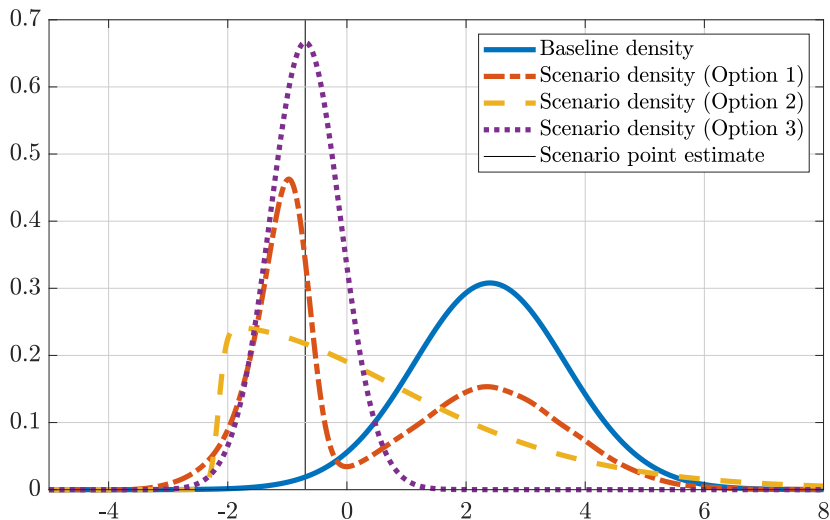
Scenarios Distributions

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1	Financial based recession	−0.6	−1.0	3.1
2	Stronger supply side	3.1	1.4	4.4
3	Greater interest rate sensitivity	1.5	0.7	3.4
4	Foreign slowdown	1.6	0.8	3.5

► Back

Scenarios Distributions



► Back

Backstop scenario

Backstop scenario \implies what if I do not trust the baseline or the scenarios?

Backstop scenario

Backstop scenario \implies what if I do not trust the baseline or the scenarios?

- The Backstop is **located at the center** of the scenarios distribution
 $\implies P50_B$ is the median of $P50_j$
- The Backstop is relatively **over-dispersed**
 $\implies P15_B = \min_{j=1,\dots,J} P15_j$
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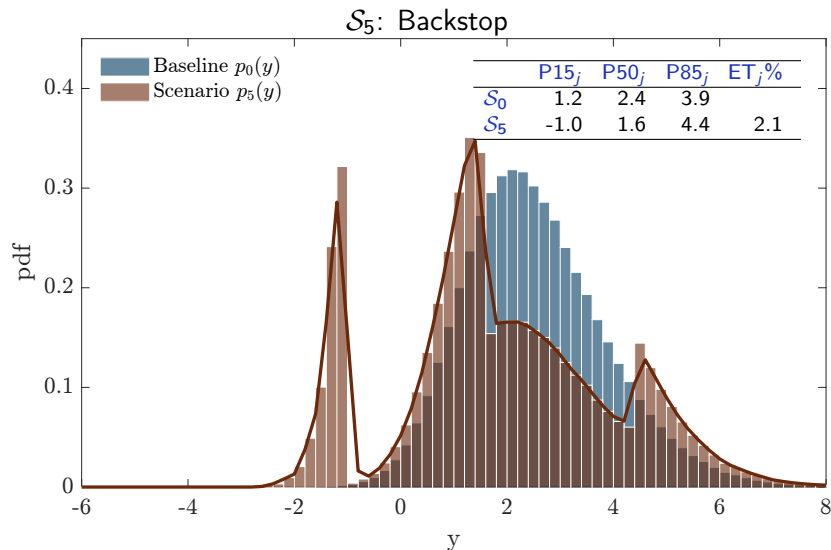
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2	Stronger supply side	3.1	1.4	4.4
3	Greater I.R. sens.	1.5	0.7	3.4
4	Foreign slowdown	1.6	0.8	3.5
5	Backstop	1.6	-1.0	4.4

► Back

Backstop scenario



► Back